SL Paper 2

Let $u = 6i + 3j + 6k$ and $v = 2i + 2j + k$.

a. Find [5]

(i) $u \bullet v$;

(ii) $|u|$;

(iii) $|v|$.

b. Find the angle between u and v. [2]

Two lines with equations $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P. Find the coordinates of P.

Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\frac{\pi}{3}$.

Find the value of k.

Let $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

a. Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

b. Let $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $\mathrm{B\hat{A}C}$. [4]

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

a(i) and (ii) (i) Write down the coordinates of A. [4]

(ii) Find the speed of the airplane in ms^{-1}.

b(i) and (ii) After seven seconds the airplane passes through a point B. [5]

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.

c. Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q\begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}$, $a \in \mathbb{R}$. [7]

The angle between the flight paths of Airplane 1 and Airplane 2 is $40°$. Find the two values of a.

Consider the points P$(2, -1, 5)$ and Q$(3, -3, 8)$. Let L_1 be the line through P and Q.

a. Show that $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$. [1]

b. The line L_1 may be represented by $\boldsymbol{r} = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$. [3]

(i) What information does the vector $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$ give about L_1?

(ii) Write down another vector representation for L_1 using $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$.

c. The point T$(-1, 5, p)$ lies on L_1. [3]

Find the value of p.

d. The point T also lies on L_2 with equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ q \end{pmatrix}$. [3]

Show that $q = -3$.

e. Let θ be the **obtuse** angle between L_1 and L_2. Calculate the size of θ. [7]

Consider the points A $(1, 5, -7)$ and B $(-9, 9, -6)$.

Let C be a point such that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

The line L passes through B and is parallel to (AC).

a. Find $\overrightarrow{AB}$. [2]

b. Find the coordinates of C. [2]

c. Write down a vector equation for L. [2]

d. Given that $\left|\overrightarrow{AB}\right| = k\left|\overrightarrow{AC}\right|$, find k. [3]

e. The point D lies on L such that $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BD}\right|$. Find the possible coordinates of D. [6]

The point O has coordinates (0 , 0 , 0) , point A has coordinates (1 , – 2 , 3) and point B has coordinates (– 3 , 4 , 2) .

a(i) and (ii). [8]

(i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $B\hat{A}O$.

b. [2]

The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1 .

c(i) and (ii). The line L_2 passes through A and is parallel to $\overrightarrow{OB}$. [6]

(i) Find a vector equation for L_2 , giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Point $C(k, -k, 5)$ is on L_2 . Find the coordinates of C.

d. [2]

The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ and passes through the point C.

Find the value of p at C.

Consider the lines L_1 and L_2 with equations $L_1 : \boldsymbol{r} = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and $L_2 : \boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P.

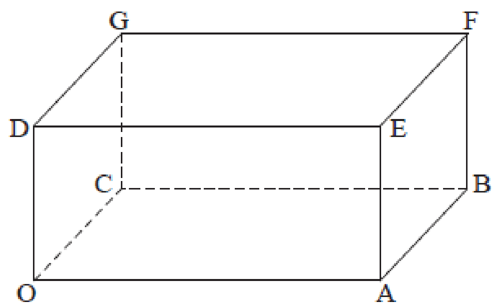
a. Find the coordinates of P. [6]

b. Show that the lines are perpendicular. [5]

c. The point $\mathrm{Q}(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2. [6]

Find the coordinates of R.

The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and $\overrightarrow{OA} = 4\boldsymbol{i}$, $\overrightarrow{OC} = 3\boldsymbol{j}$, $\overrightarrow{OD} = 2\boldsymbol{k}$.



a(i), (ii) and (iii). (i) Find $\overrightarrow{OB}$. [5]

(ii) Find $\overrightarrow{OF}$.

(iii) Show that $\overrightarrow{AG} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$.

b(i) and (ii). Write down a vector equation for [4]

(i) the line OF;

(ii) the line AG.

c. Find the obtuse angle between the lines OF and AG. [7]

Consider the points A(5, 2, 1) , B(6, 5, 3) , and C(7, 6, $a + 1$) , $a \in \mathbb{R}$.

Let q be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

a. Find [3]

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AC}$.

b. Find the value of a for which q $= \frac{\pi}{2}$. [4]

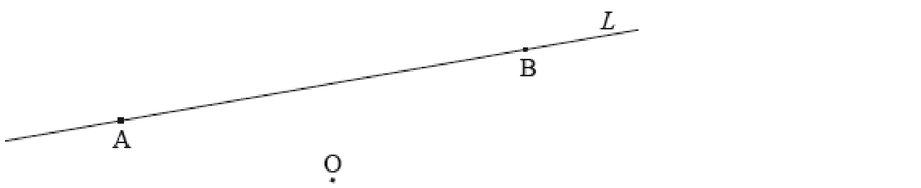
c. i. Show that $\cos q = \frac{2a+14}{\sqrt{14a^2+280}}$. [8]

ii. Hence, find the value of a for which q $= 1.2$.

c.ii. Hence, find the value of a for which q $= 1.2$. [4]

Let $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. The vector $\boldsymbol{v} + p\boldsymbol{w}$ is perpendicular to $\boldsymbol{w}$. Find the value of p.

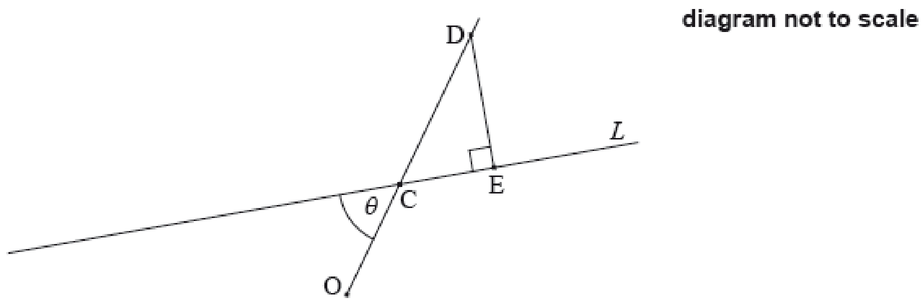
The points A and B lie on a line L, and have position vectors $\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$ respectively. Let O be the origin. This is shown on the following diagram.



The point C also lies on L, such that $\overrightarrow{\mathrm{AC}} = 2\overrightarrow{\mathrm{CB}}$.

Let θ be the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{OC}}$.

Let D be a point such that $\overrightarrow{\mathrm{OD}} = k\overrightarrow{\mathrm{OC}}$, where $k > 1$. Let E be a point on L such that $\mathrm{C\hat{E}D}$ is a right angle. This is shown on the following diagram.



a. Find $\overrightarrow{\mathrm{AB}}$. [2]

b. Show that $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$. [[N/A

c. Find θ. [5]

d. (i) Show that $\left|\overrightarrow{\mathrm{DE}}\right| = (k-1)\left|\overrightarrow{\mathrm{OC}}\right| \sin\theta$. [6]

(ii) The distance from D to line L is less than 3 units. Find the possible values of k.

Line L_1 passes through points $\mathrm{A}(1, -1, 4)$ and $\mathrm{B}(2, -2, 5)$.

Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

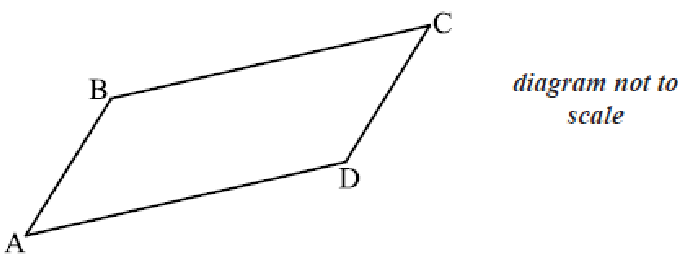
a. Find $\overrightarrow{\mathrm{AB}}$. [2]

b. Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. [2]

c. Find the angle between L_1 and L_2 . [7]

d. The lines L_1 and L_2 intersect at point C. Find the coordinates of C. [6]

The diagram shows a parallelogram ABCD.



The coordinates of A, B and D are A(1, 2, 3) , B(6, 4,4) and D(2, 5, 5) .

a(i), (ii) and (iii). [5]

(i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$.

(ii) Find $\overrightarrow{\mathrm{AD}}$.

(iii) **Hence** show that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$.

b. Find the coordinates of point C. [3]

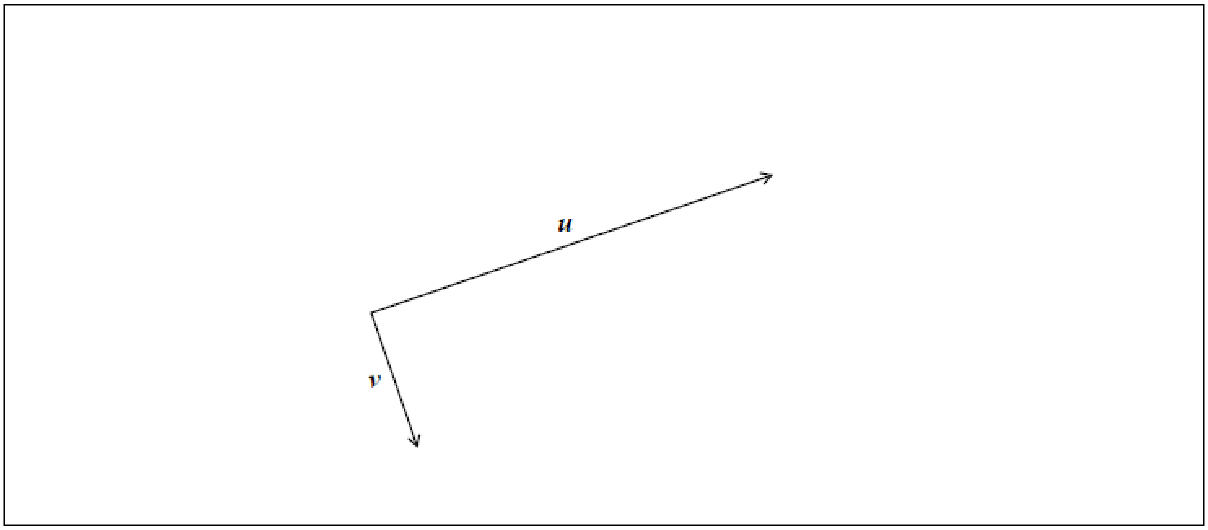
c(i) and (ii). [7]

(i) Find $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AD}}$.

(ii) **Hence** find angle A.

d. Hence, or otherwise, find the area of the parallelogram. [3]

The following diagram shows two perpendicular vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.



a. Let $w = u - v$. Represent w on the diagram above. [2]

b. Given that $u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find \(n\). [4]

Consider the lines L_1 , L_2 , L_2 , and L_4 , with respective equations.

$$L_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$$L_2 \ : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + p \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$L_3 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix}$$

$$L_4 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = q \begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$$

a. Write down the line that is parallel to L_4 . [1]

b. Write down the position vector of the point of intersection of L_1 and L_2 . [1]

c. Given that L_1 is perpendicular to L_3 , find the value of a . [5]

Two points P and Q have coordinates (3, 2, 5) and (7, 4, 9) respectively.

Let $\vec{PR} = 6\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

a.i. Find $\vec{PQ}$. [2]

a.ii. Find $\left|\vec{PQ}\right|$. [2]

b. Find the angle between PQ and PR. [4]

c. Find the area of triangle PQR. [2]

d. Hence or otherwise find the shortest distance from R to the line through P and Q. [3]

The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.
